1 Solutions

1.4.25

$$\begin{aligned}
& \frac{2x^2+3x+1}{x^2+2x-15} \div \frac{x^2+6x+5}{2x^2-7x+3} \\
= & \frac{2x^2+3x+1}{x^2+2x-15} \times \frac{2x^2-7x+3}{x^2+6x+5} \\
= & \frac{(2x+1)(x+1)}{(x-3)(x+5)} \times \frac{(2x-1)(x-3)}{(x+1)(x+5)} \\
= & \frac{(2x+1)(x+1)(2x-1)(x-3)}{(x-3)(x+5)(x+1)(x+5)} \\
= & \frac{(2x+1)(2x-1)}{(x+5)(x+5)} \\
= & \frac{(2x+1)(2x-1)}{(x+5)^2}
\end{aligned}$$

1.4.27

$$\begin{aligned}
& \frac{\frac{x^2}{x+1}}{\frac{x}{x^2+2x+1}} \\
= & \frac{x^2}{x+1} \div \frac{x}{x^2+2x+1} \\
= & \frac{x^2}{x+1} \times \frac{x^2+2x+1}{x+1} \\
= & \frac{x^2}{x+1} \times \frac{(x+1)^2}{x+1} \\
= & \frac{(x^2)(x+1)^2}{(x+1)^2} \\
= & x^2
\end{aligned}$$

1.4.34

$$\frac{1}{x+1} - \frac{1}{x-1} = \frac{x-1}{(x+1)(x-1)} - \frac{x+1}{(x+1)(x-1)} = \frac{(x-1)+(x+1)}{(x+1)(x-1)} = \frac{2x}{(x+1)(x-1)}$$

1.4.38

$$\frac{5}{2x-3} - \frac{3}{(2x-3)^2} = \frac{5(2x-3)}{(2x-3)^2} - \frac{3}{(2x-3)^2} = \frac{5(2x-3)-3}{(2x-3)^2} = \frac{10x-9-3}{(2x-3)^2} = \frac{10x-12}{(2x-3)^2}$$

1.4.43

$$\frac{2}{x+3} - \frac{1}{x^2+7x+12} = \frac{2}{x+3} - \frac{1}{(x+3)(x+4)} = \frac{2(x+4)}{(x+3)(x+4)} - \frac{1}{(x+3)(x+4)}$$

$$= \frac{2(x+4)-1}{(x+3)(x+4)} = \frac{2x+8-1}{(x+3)(x+4)} = \frac{2x+7}{(x+3)(x+4)}$$

1.4.47

$$\frac{2}{x}+\frac{3}{x-1}-\frac{4}{x^2-x}=\frac{2}{x}+\frac{3}{x-1}-\frac{4}{x(x-1)}=\frac{2(x-1)}{x(x-1)}+\frac{3x}{x(x-1)}-\frac{4}{x(x-1)}$$
$$=\frac{2(x-1)+3x-4}{x(x-1)}=\frac{2x-2+3x-4}{x(x-1)}=\frac{5x-6}{x(x-1)}$$

1.4.51

$$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{x^2}-\frac{1}{y^2}}=\frac{\frac{x^2}{xy}-\frac{y^2}{xy}}{\frac{y^2}{x^2y^2}-\frac{x^2}{x^2y^2}}=\frac{\frac{x^2-y^2}{xy}}{\frac{y^2-x^2}{x^2y^2}}=\frac{x^2-y^2}{xy}\div\frac{y^2-x^2}{x^2y^2}=\frac{x^2-y^2}{xy}\times\frac{x^2y^2}{y^2-x^2}$$
$$=\frac{(x+y)(x-y)}{xy}\times\frac{x^2y^2}{(y-x)(y+x)}=\frac{(x+y)(x-y)(x^2y^2)}{(xy)(y+x)(y-x)}=\frac{(x+y)(x-y)(x^2y^2)}{(xy)(x+y)((-1)(x-y))}=-xy$$

1.4.58

$$\frac{x^{-1}+y^{-1}}{(x+y)^{-1}}=(x^{-1}+y^{-1})(x+y)=x^{-1}x+x^{-1}y+y^{-1}x+y^{-1}y=1+\frac{y}{x}+\frac{x}{y}+1=2+\frac{x}{y}+\frac{y}{x}$$
$$=\frac{2xy}{xy}+\frac{x^2}{xy}+\frac{y^2}{xy}=\frac{2xy+x^2+y^2}{xy}=\frac{(x+y)^2}{xy}$$

1.4.63

$$\begin{aligned}
&\frac{\frac{1-(x+h)}{2+(x+h)}-\frac{1-x}{2+x}}{h}\\
&=\frac{\frac{1-x-h}{2+x+h}-\frac{1-x}{2+x}}{h}\\
&=\frac{\frac{(1-x-h)(2+x)}{(2+x+h)(2+x)}-\frac{(1-x)(2+x+h)}{(2+x+h)(2+x)}}{h}\\
&=\frac{\frac{(1-x-h)(2+x)-(1-x)(2+x+h)}{(2+x+h)(2+x)}}{h}\\
&=\frac{(1-x-h)(2+x)-(1-x)(2+x+h)}{(2+x+h)(2+x)}\div h\\
&=\frac{(1-x-h)(2+x)-(1-x)(2+x+h)}{(2+x+h)(2+x)}\times\frac{1}{h}\\
&=\frac{2+x-2x-x^2-2h-xh-2-x-h+2x+x^2+xh}{(2+x+h)(2+x)}\times\frac{1}{h}\\
&=\frac{(-x^2+x^2)+(x-x)+(-xh+xh)+(-2h-h)+(2-2)}{(2+x+h)(2+x)}\times\frac{1}{h}\\
&=-\frac{3h}{(2+x+h)(2+x)}\times\frac{1}{h}\\
&=-\frac{3}{(2+x+h)(2+x)}
\end{aligned}$$

1.4.71

$$\begin{aligned}
& \frac{3(1+x)^{1/3} - x(1+x)^{-2/3}}{(1+x)^{2/3}} \\
= & \frac{(1+x)^{-2/3}(3(1+x) - x)}{(1+x)^{2/3}} \\
= & \frac{(1+x)^{-2/3}(3+3x-x)}{(1+x)^{2/3}} \\
= & \frac{(1+x)^{-2/3}(2x-3)}{(1+x)^{2/3}} \\
= & \frac{2x-3}{(1+x)^{4/3}}
\end{aligned}$$

1.4.80

$$\frac{\sqrt{3}+\sqrt{5}}{2} = \frac{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})}{2(\sqrt{3}-\sqrt{5})} = \frac{3-5}{2(\sqrt{3}-\sqrt{5})} = -\frac{2}{2(\sqrt{3}-\sqrt{5})} = -\frac{1}{\sqrt{3}-\sqrt{5}} = \frac{1}{\sqrt{5}-\sqrt{3}}$$

1.4.81

$$\frac{\sqrt{r}+\sqrt{2}}{5} = \frac{(\sqrt{r}+\sqrt{2})(\sqrt{r}-\sqrt{2})}{5(\sqrt{r}-\sqrt{2})} = \frac{r-2}{5(\sqrt{r}-\sqrt{2})}$$

1.4.87 It is false for $x = 4$.

1.4.89 It is false for $x = 2$ and $y = 1$.

1.4.92

$$\frac{1+x+x^2}{x} = \frac{1}{x} + \frac{x}{x} + \frac{x^2}{x} = \frac{1}{x} + 1 + x$$

1.5.1

$$\begin{aligned}
4x + 7 &= 9x - 3 \\
7 &= 9x - 4x - 3 \\
7 + 3 &= 9x - 4x \\
10 &= 5x \\
2 &= x
\end{aligned}$$

In conclusion, (a) is false, and (b) is true.

1.5.3

$$\begin{aligned}
\frac{1}{x} - \frac{1}{x-4} &= 1 \\
\frac{x-4}{x(x-4)} - \frac{x}{x(x-4)} &= 1 \\
\frac{x-4-x}{x(x-4)} &= 1 \\
-\frac{4}{x(x-4)} &= 1 \\
-4 &= x(x-4) \\
-4 &= x^2 - 4x \\
0 &= x^2 - 4x + 4 \\
0 &= (x-2)^2 \\
x &= 2
\end{aligned}$$

In conclusion, (a) is true, and (b) is false.

1.5.13

$$\begin{aligned}
2(1-x) &= 3(1+2x)+5 \\
2-2x &= 3+6x+5 \\
-2x-6x &= 3+5-2 \\
-8x &= 6 \\
x &= -\frac{3}{4}
\end{aligned}$$

1.5.15

$$\begin{aligned}
x - \frac{1}{3}x - \frac{1}{2}x - 5 &= 0 \\
\left(1 - \frac{1}{3} - \frac{1}{2}\right)x - 5 &= 0 \\
\frac{1}{6}x - 5 &= 0 \\
\frac{1}{6}x &= 5 \\
x &= 30
\end{aligned}$$

1.5.16

$$\begin{aligned}
2x - \frac{x}{2} + \frac{x+1}{4} &= 6x \\
\frac{8x}{4} - \frac{2x}{4} + \frac{x+1}{4} &= 6x \\
\frac{8x - 2x + x + 1}{4} &= 6x \\
\frac{7x+1}{4} &= 6x \\
7x+1 &= 24x \\
1 &= 17x \\
\frac{1}{17} &= x
\end{aligned}$$

1.5.25

$$\begin{aligned}
\frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\
\frac{1}{R} - \frac{1}{R_2} &= \frac{1}{R_1} \\
\frac{R_2}{RR_2} - \frac{R}{R_2} &= \frac{1}{R_1} \\
\frac{R_2 - R}{RR_2} &= \frac{1}{R_1} \\
R_1\left(\frac{R_2 - R}{RR_2}\right) &= 1 \\
R_1 &= \frac{RR_2}{R_2 - R}
\end{aligned}$$

1.5.27

$$\begin{aligned}
\frac{ax+b}{cx+d} &= 2 \\
ax+b &= 2(cx+d) \\
ax+b &= 2cx+2d \\
ax-2cx &= 2d-b \\
(a-2c)x &= 2d-b \\
x &= \frac{2d-b}{a-2c}
\end{aligned}$$

1.5.34

$$\begin{aligned}
A &= P\left(1+\frac{i}{100}\right)^2 \\
\frac{A}{P} &= \left(1+\frac{i}{100}\right)^2 \\
\pm\sqrt{\frac{A}{P}} &= 1+\frac{i}{100} \\
\pm\sqrt{\frac{A}{P}} - 1 &= \frac{i}{100} \\
\pm 100\sqrt{\frac{A}{P}} - 100 &= i
\end{aligned}$$

1.5.42 $2y^2 + 7y + 3 = (2y+1)(y+3) = 0$, hence $y = -\frac{1}{2}$ and $y = -3$.

1.5.49

$$\begin{aligned}
2x^2+8x+1 &= 0\\
2x^2+8x &= -1\\
2(x^2+4x) &= -1\\
2(x^2+4x+4) &= -1+8\\
2(x^2+4x+4) &= 7\\
2(x+2)^2 &= 7\\
(x+2)^2 &= \frac{7}{2}\\
x+2 &= \pm\sqrt{\frac{7}{2}}\\
x &= -2\pm\sqrt{\frac{7}{2}}\\
x &= -2\pm\frac{\sqrt{14}}{2}
\end{aligned}$$

1.5.55

$$x=\frac{-6\pm\sqrt{3^2-4\cdot 1\cdot 1}}{2}=\frac{-6\pm\sqrt{5}}{2}=-3\pm\frac{\sqrt{5}}{2}$$

1.5.57 $2x^2+x-3=(2x+3)(x-1)=0$, hence $x=-\frac{3}{2}$ and $x=1$.

1.5.75

$$\begin{aligned}
\frac{1}{x-1}+\frac{1}{x+2} &= \frac{5}{4}\\
\frac{x+2}{(x-1)(x+2)}+\frac{x-1}{(x-1)(x+2)} &= \frac{5}{4}\\
\frac{x+2+x-1}{(x-1)(x+2)} &= \frac{5}{4}\\
\frac{2x+1}{(x-1)(x+2)} &= \frac{5}{4}\\
2x+1 &= \frac{5}{4}(x-1)(x+2)\\
2x+1 &= \frac{5}{4}(x^2+x-2)\\
2x+1 &= \frac{5}{4}x^2+\frac{5}{4}x-\frac{5}{2}\\
-\frac{5}{4}x^2+\left(2-\frac{5}{4}\right)x+\left(1+\frac{5}{2}\right) &= 0\\
-\frac{5}{4}x^2+\frac{3}{4}x+\frac{7}{2} &= 0\\
-5x^2+3x+14 &= 0\\
5x^2-3x-14 &= 0\\
(5x+7)(x-2) &= 0\\
x &= -\frac{7}{5},2
\end{aligned}$$

1.5.79

$$\begin{aligned}
\frac{x+5}{x-2} &= \frac{5}{x+2}+\frac{28}{x^2-4}\\
\frac{x+5}{x-2}-\frac{5}{x+2}-\frac{28}{x^2-4} &= 0\\
\frac{(x+5)(x+2)}{(x-2)(x+2)}-\frac{5(x-2)}{(x-2)(x+2)}-\frac{28}{(x-2)(x+2)} &= 0\\
\frac{(x+5)(x+2)-5(x-2)-28}{(x-2)(x+2)} &= 0\\
\frac{x^2+7x+10-5x+10-28}{(x-2)(x+2)} &= 0\\
\frac{x^2+2x-8}{(x-2)(x+2)} &= 0\\
\frac{(x-2)(x+4)}{(x-2)(x+2)} &= 0\\
\frac{x+4}{x+2} &= 0\\
x+4 &= 0\\
x &= -4
\end{aligned}$$

1.5.80

$$\begin{aligned}
\frac{x}{2x+7}-\frac{x+1}{x+3} &= 1\\
\frac{x(x+3)}{(2x+7)(x+3)}-\frac{(x+1)(2x+7)}{(2x+7)(x+3)} &= 1\\
\frac{x(x+3)-(x+1)(2x+7)}{(2x+7)(x+3)} &= 1\\
\frac{x^2+3x-2x^2-7x-2x-7}{(2x+7)(x+3)} &= 1\\
\frac{(1-2)x^2+(3-7-2)x-7}{(2x+7)(x+3)} &= 1\\
\frac{-x^2-6x-7}{(2x+7)(x+3)} &= 1\\
-x^2-6x-7 &= (2x+7)(x+3)\\
-x^2-6x-7 &= 2x^2+7x+6x+21\\
0 &= 2x^2+7x+6x+21+x^2+6x+7\\
0 &= (2+1)x^2+(7+6+6)x+(21+7)\\
0 &= 3x^2+19x+28\\
0 &= (x+4)(3x+7)\\
x &= -4,-\frac{7}{3}
\end{aligned}$$

1.5.82

$$\begin{aligned}
\sqrt{5-x}+1 &= x-2 \\
\sqrt{5-x} &= x-2-1 \\
\sqrt{5-x} &= x-3 \\
5-x &= (x-3)^2 \\
5-x &= x^2-6x+9 \\
0 &= x^2-6x+9-5+x \\
0 &= x^2+(-6+1)x+(9-5) \\
0 &= x^2-5x+4 \\
0 &= (x-1)(x-4) \\
x &= 1,4
\end{aligned}$$

We check for extraneous solutions:

1. $\sqrt{5-1}+1=4+1=5\neq 3=5-2$, hence $x=1$ is an extraneous solution.
2. $\sqrt{5-4}+1=1+1=2=4-2$, hence $x=4$ is not an extraneous solution.

We conclude that $x=4$.

1.5.83

$$\begin{aligned}
2x+\sqrt{x+1} &= 8 \\
\sqrt{x+1} &= 8-2x \\
x+1 &= (8-2x)^2 \\
x+1 &= 4x^2-32x+64 \\
0 &= 4x^2-32x+64-x-1 \\
0 &= 4x^2+(-32-1)x+(64-1) \\
0 &= 4x^2-33x+63 \\
0 &= (x-3)(4x-21) \\
x &= 3, \frac{21}{4}
\end{aligned}$$

We check for extraneous solutions:

1. $2\cdot 3+\sqrt{3+1}=6+2=8$, hence $x=3$ is not an extraneous solution.
2. $2\cdot\frac{21}{4}+\sqrt{\frac{21}{4}+1}=\frac{21}{2}+\frac{\sqrt{21}}{2}\neq 8$, hence $x=\frac{21}{4}$ is an extraneous solution.

We conclude that $x=3$.

1.5.86

$$\begin{aligned}
x^4-5x^2+4 &= 0 \\
(x^2-1)(x^2-4) &= 0 \\
(x+1)(x-1)(x-2)(x+2) &= 0 \\
x &= 1,-1,2,-2
\end{aligned}$$

1.5.96 We consider two cases.

Case 1. We assume that $3x+5 \geq 0$. Then:

$$\begin{aligned} 3x+5 &= 1 \\ 3x &= 1-5 \\ 3x &= -4 \\ x &= -\frac{4}{3} \end{aligned}$$

Case 2. We assume that $3x+5 < 0$. Then:

$$\begin{aligned} -(3x+5) &= 1 \\ 3x+5 &= -1 \\ 3x &= -1-5 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$

We conclude that $x = -\frac{4}{3}, -2$.

Note: Alternatively, we could square both sides, as $|A|^2 = A^2$ for all A.

1.5.97 We consider two cases.

Case 1. We assume that $x-4 \geq 0$. Then:

$$\begin{aligned} x-4 &= 0.01 \\ x &= 0.01+4 \\ x &= 4.01 \end{aligned}$$

Case 2. We assume that $x-4 < 0$. Then:

$$\begin{aligned} -(x-4) &= 0.01 \\ x-4 &= -0.01 \\ x &= -0.01+4 \\ x &= 3.99 \end{aligned}$$

We conclude that $x = 4.01, 3.99$.

1.5.112 The conditions given in the problem prescribe that the sphere, cylinder, and cone all have the same volume. Hence, verifying (a) is a simple matter of setting the volume formulas to be equal. The first equation, solved for h_1, is:

$$\begin{aligned} \frac{4}{3}\pi r^3 &= \pi r^2 h_1 \\ \frac{4}{3}r &= h_1 \end{aligned}$$

The second equation, solved for h_2, is:

$$\begin{aligned} \frac{4}{3}\pi r^3 &= \frac{1}{3}\pi r^2 h_2 \\ 4r &= h_2 \end{aligned}$$

2 Grading

1.4.25 should be a straightforward exercise in dividing rational expressions. **1.4.27** may seem a bit trickier, but it is, in fact, just as straightforward.

1.4.63 is a (somewhat) tedious exercise in simplifying compound fractions. We could view it as a combined exercise in dividing rational expressions (**1.4.25**, **1.4.27**) and adding rational expressions (**1.4.34**, **1.4.38**, **1.4.43**, **1.4.47**). Similar problems include **1.4.51**, **1.4.58**, and **1.4.71**.

1.5.49 deals with the method of completingthe square. See *Supplementary Note: Week 4* if you need additional practice.

1.5.80 is a typical problem in equations involving (relatively) simple fractions. Note that the process of checking for the extraneous solutions is implicit in the solution; it is a simple matter of glancing at the original denominators to check if the solutions would cause a singularity.

1.5.82 is a nice exercise in solving equations involving a radical. Once you learn the little trick of isolating the square root, there should be little difficulty in dealing with this sort of problems.

1.5.96 is a standard exercise in solving absolute value equations. I could have graded **1.5.97**, but...no. (Come on now, all **1.5.97** is asking you is *to find two numbers 0.01 away from 4 on the number line.*)